

NAME:

Grade:

Start	1	2	3	4	5	6	7	8	9	10

Combinatorics

Written examination

23 November 2020

Choose or fill in the correct answers:

Start: 1p

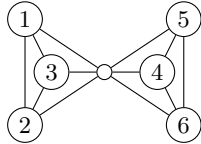
- (1p) How many different strings can be produced by rearranging the letters of BETHLEHEM?
(a) 60480 (b) 181440 (c) 30240 (d) 59049 (e) (f) -----
- (0.5p) Which is the next permutation of $\langle 1, 5, 2, 3, 6, 4 \rangle$?

- (0.5p) Compute the 8-permutation with repetition of the set $\{1, 2, 3, 4, 5\}$ with rank 40 in the lexicographic ordering.

- (1p) Which is the rank of the permutation $\langle 3, 1, 5, 2, 4 \rangle$ in lexicographic order?

- (1p)
 - How many 2-permutations has the set $\{1, 2, 3, 4\}$?
 - How many 2-permutations with repetition has the set $\{1, 2, 3, 4\}$?
 - How many subsets with 2 elements has the set $\{1, 2, 3, 4\}$?
 - How many subsets with at most 2 elements has the set $\{1, 2, 3, 4\}$?(a): ----- (b): ----- (c): ----- (d): -----
- (1p) Solve the following recurrence relation together with the initial conditions given:
 $a_n = 5 a_{n-1} - 6 a_{n-2}$, for $n \geq 2$, $a_0 = 1$, $a_1 = 0$.
 $a_n =$ -----
- (1p) In how many ways 5 persons can be seated at a round table?
(a) 120 (b) 24 (c) 12 (d) 32

8. (1p) In how many ways can be colored the following configuration by using colors from the set {red, yellow, blue}?



Knowing that the group of symmetries $G = \{(1)(2)(3)(4)(5)(6), (1, 2)(3)(4)(5, 6), (1, 5)(2, 6)(3, 4), (1, 6)(2, 5)(3, 4)\}$

- (a) 64 (b) 48 (c) 128 (d) 18 (e) 216
9. (1p)
- (a) Which of the following lists is a valid permutation type:
- (a) [1, 1, 0, 1] (b) [1, 1, 0, 0]
(c) [1, 0, 1, 0, 0] (d) [0, 0, 0, 0, 1]
(e) [0, 1, 0, 1, 0, 0] (f) [1, 0, 1, 0, 0, 0]
- (b) How many permutations have the same type as the permutation $\langle 1, 3, 2, 5, 4 \rangle$?
- (a): _____ (b): _____
10. (1p) How many ways are there to choose 6 coins from a pocket if there are 6 coins of 1 cent, 6 coins of 5 cents, 6 coins of 10 cents, and 6 coins of 50 cents?
- (a) 84 (b) 320 (c) 120 (d) 6 (e) 210